Probing exotic matter in neutron star cores with g-mode oscillations

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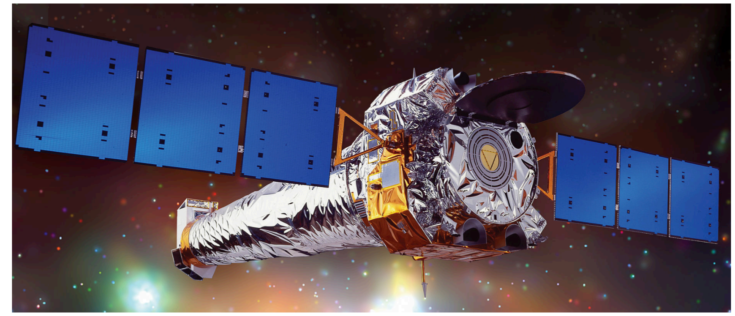
@Institute of Physics Academia Sinica

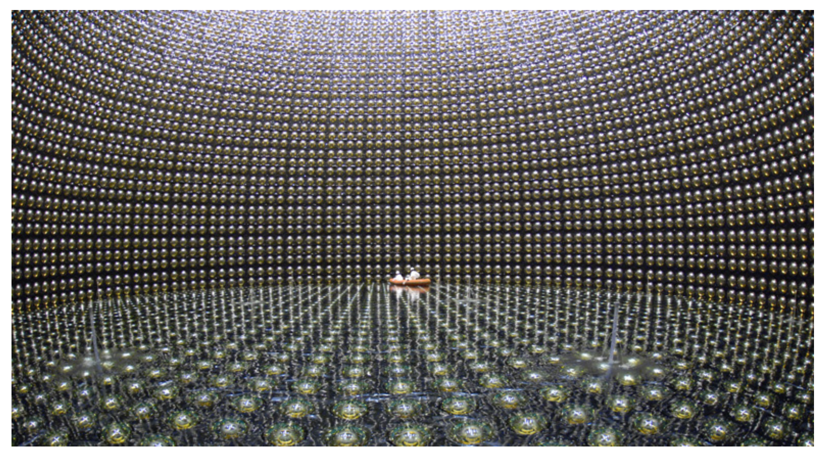
Multi-messenger era



Photons

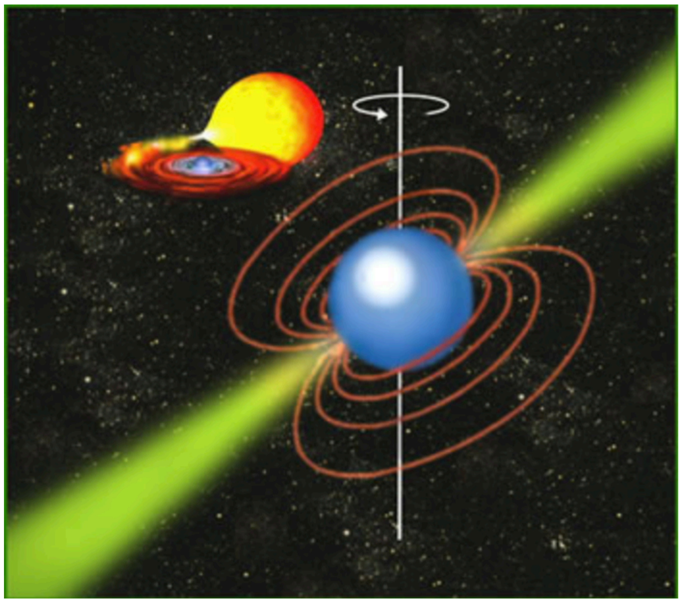
Gravitational Waves

Neutrinos



Motivation

most compact objects
second only to black holes



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- still far from understanding NSs' composition after half a century since their discovery
- NS mass-radius <-> pressure vs. energy density (EoS): important, but **not enough**

main sources for LIGO/Virgo:

- NS, BH binary mergers
- supernovae, NS/BH formation
- spinning NSs in X-ray binaries
- isolated NSs: instabilities, deformations

WDs: $M/R \sim 10^{-4}$
NSs: $M/R \sim 0.2$
BHs: $M/R = 0.5$

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\,\mathrm{km})$	$\gtrsim \rho_{\mathrm{nuclear}}$	$\sim 30\,\mathrm{MeV}$

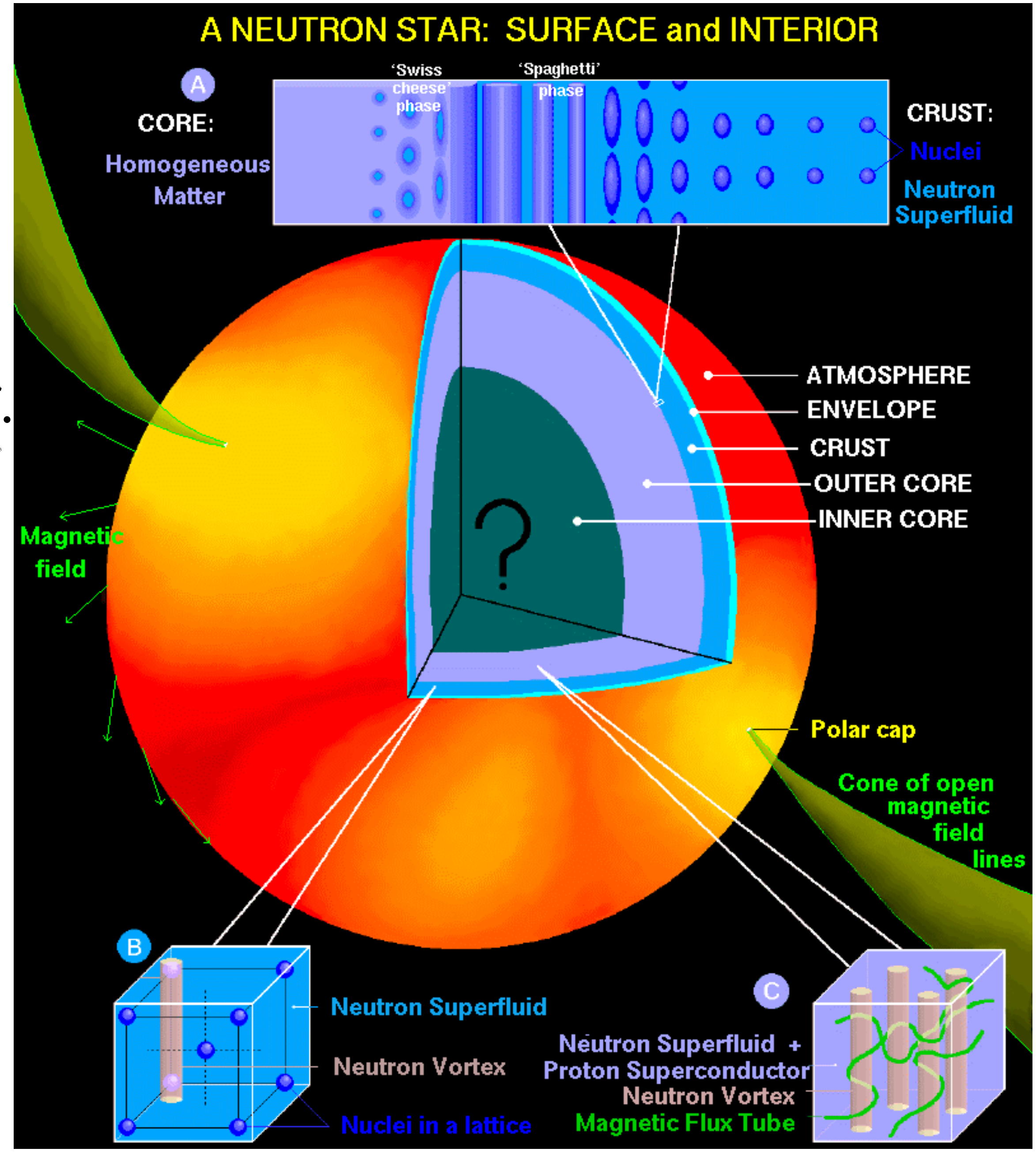
credit: Dany Page

Dense matter in NSs

- stable nuclei
- neutron-rich nuclei
- neutron-rich nuclei with quasi-free neutrons

. . . .

- homogeneous nucleonic matter
- exotica



Fundamental questions

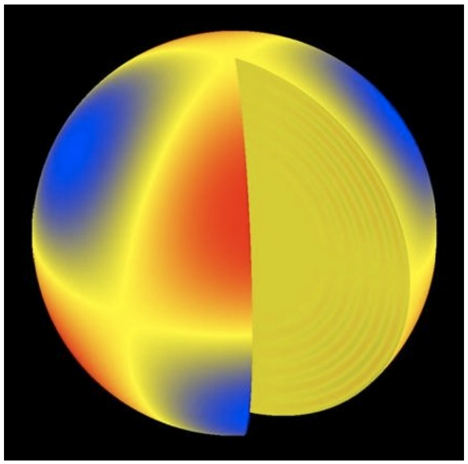
- what are the most relevant lower-energy degrees of freedom?
- how does deconfinement evolve as T->0 on the QCD phase diagram?

Stellar oscillations

matter imprints in **transient** GWs

- tidal effects on pre-merger (inspiral) waveform of BNS mergers
- tidal disruption in NS-BH mergers
- oscillations of merger remnant
- oscillation in supernova post-collapse phase

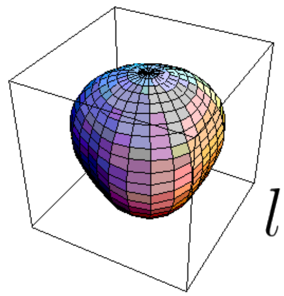
©NASA/Kepler



..unless they become unstable

oscillation modes ("ringing") -> **continuous** GWs

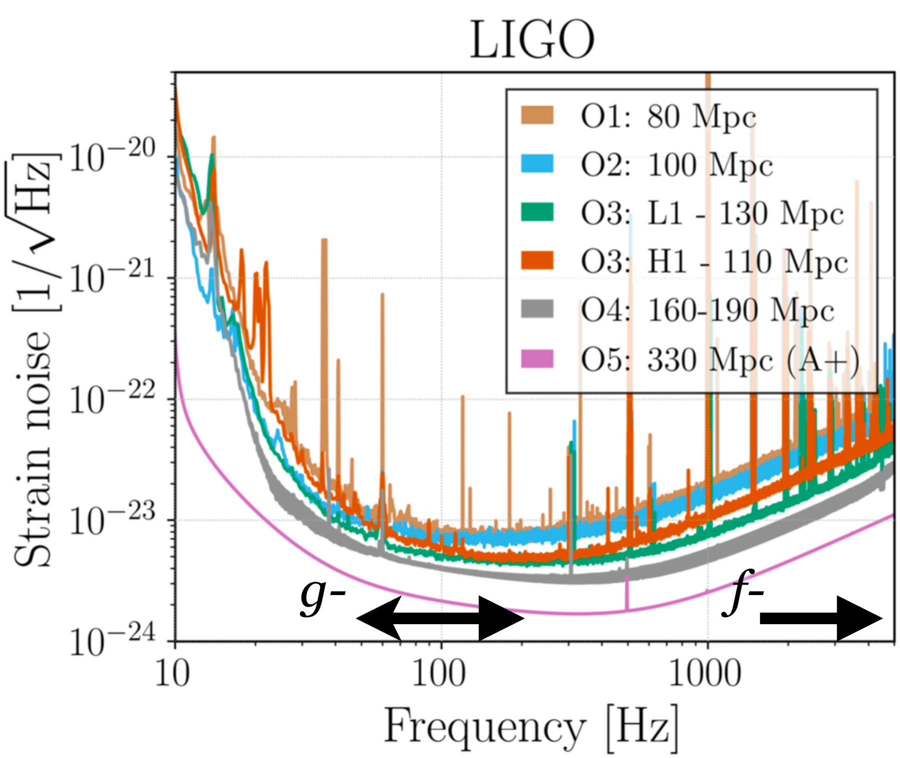
- f-mode (fundamental mode) scales with average density
- p-mode (pressure mode) probes the sound speed
- g-mode (gravity mode) sensitive to **composition**/thermal gradients
- w-mode, s-mode, i-mode/r-mode..



$l = 3, \; m = 3$

small amplitude oscillations -> weak (continuous) emission of GWs

Restoring forces and frequency



oscillation modes ("ringing") -> **continuous** GWs

- p-mode/f-mode: main restoring force is the pressure (>1.5 kHz)

$$\nu \approx \sqrt{\frac{GM}{R^3}}$$

- inertial modes (r-modes): main restoring force is the Coriolis force

$$\nu \approx \Omega$$

- w-modes: pure space-time modes i.e. only in GR (>5 kHz)

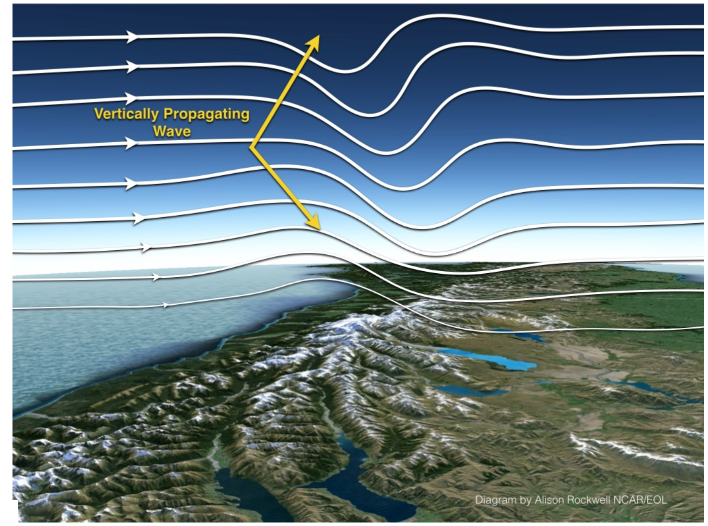
$$\nu \approx \frac{1}{R}\left(\frac{GM}{Rc^2}\right)$$

- shear-/torsional-; many other more

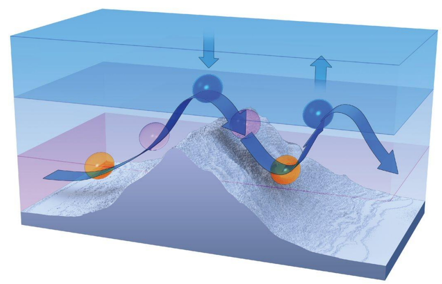
g-modes (gravity modes)

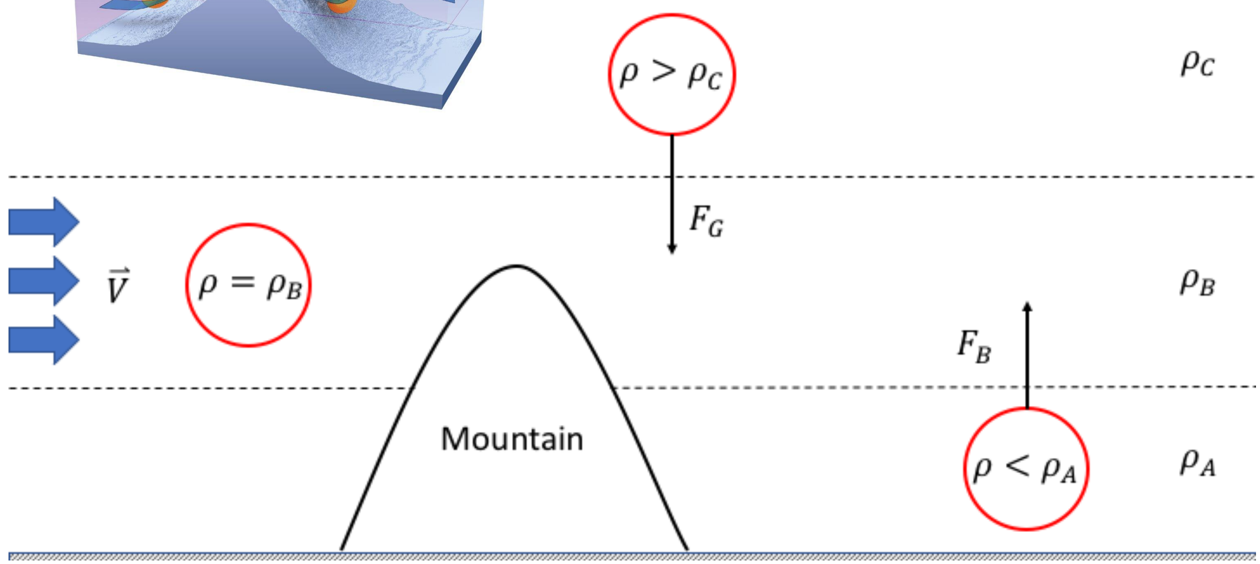
restoring forces from buoyancy/gravity

- e.g. atmospheric/ocean waves
- hydrostatic equilibrium: gravitational force balanced by pressure gradient force
- perturbed from equilibrium -> gravity or buoyancy pulls/pushes it back -> oscillation



AUSTRALIAN ANTARCTIC PROGRAM





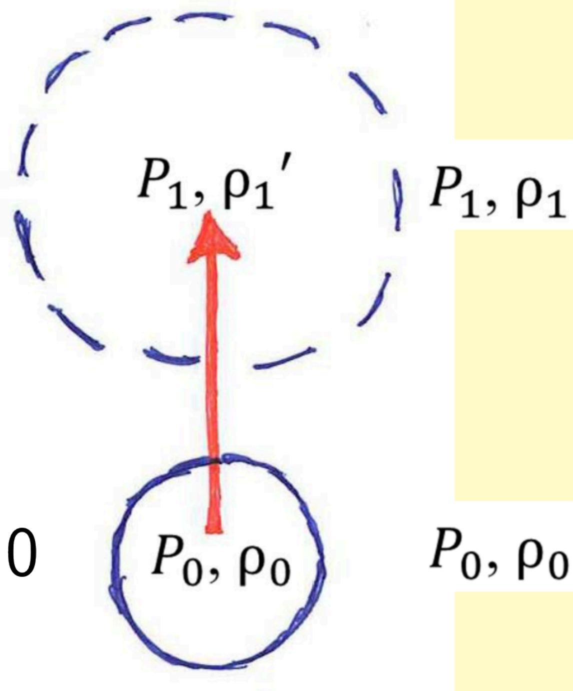
Brunt–Väisälä (buoyancy) frequency

- pressure instantaneously equilibrated, but not for composition and density
- continuity equation & the equation of motion
- “adiabatic” (composition frozen) sound speed vs. “equilibrium” sound speed

$$\Delta\rho = -\rho\frac{\partial^2\xi}{\partial t^2}$$

$$\downarrow \vec{g}$$

$$\frac{\partial^2\xi}{\partial t^2} + N^2\xi = 0$$



credit: Andreas Reisenegger

$$dp/dr = -\rho g$$

(hydrostatic equilibrium)

$$N^2 \equiv g^2\left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2}\right) e^{\nu-\lambda}$$

← local metric coefficients

$$c_{eq}^2 = \left(\frac{dp/dr}{d\varepsilon/dr}\right) \qquad c_{ad}^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{y_i}$$

NS core g-modes

$$N^2 \equiv g^2 \left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2} \right) e^{\nu - \lambda}$$

- stability criterion: $N^2 > 0$ -> stable stratification
- assuming cold NS (zero temperature/ entropy); no convection or turbulence

$$\text{osc. amplitude} \sim e^{i\omega t},\ \omega \propto \sqrt{N^2}$$

$$c_{ad}^2 \geqslant c_{eq}^2 \Rightarrow \text{mode stablized}$$

- bulk region of the NS liquid core; restored by buoyancy due to the chemical composition gradient e.g. proton fraction
- crustal modes behave differently and are expected to be very small
- e.g. in n-p-e matter

$$c_{ad}^2 - c_{eq}^2 = - \left(\frac{\partial p}{\partial Y_e} \right)_{\varepsilon} \left(\frac{dY_e}{d\varepsilon} \right)$$

Sensitivity to composition

(arXiv:2004.08293)

$$E(u,x) = E_{\rm SNM} + S_2(u)(1-2x)^2 + ..$$

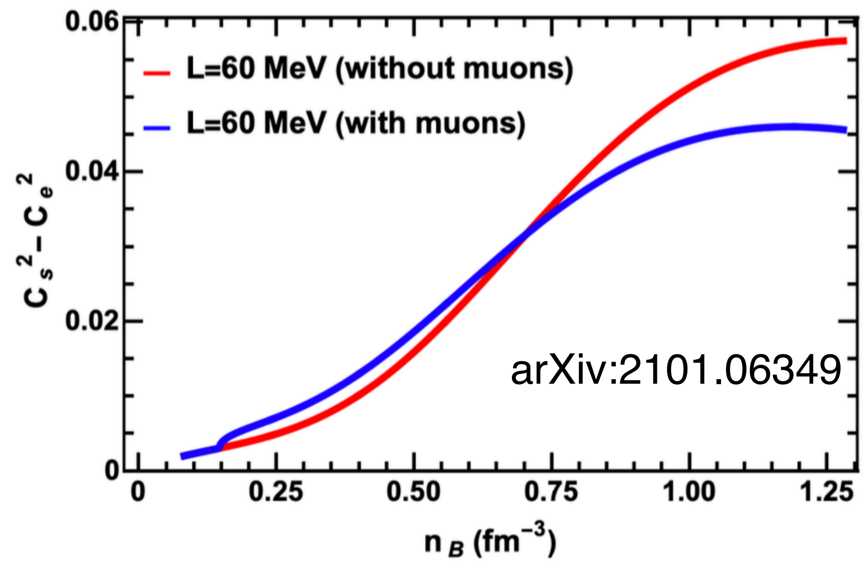
$$u = n_B/n_{sat},\ n_{sat} \approx 0.16\,{\rm fm}^{-3}$$

$$x \equiv y_p = n_p/n_B$$

$$c_{ad}^2 - c_{eq}^2 = \frac{n_B^2\left[\left(\frac{\partial\tilde{\mu}}{\partial n_B}\right)_x\right]^2}{\mu_n\left(\frac{\partial\tilde{\mu}}{\partial x}\right)_{n_B}}$$

$$\tilde{\mu} = \mu_e + \mu_p - \mu_n$$

- e.g. "ZL" EoS parametrization using symmetry energy slope
- $L = 3n_{\rm B}dS_2/dn_{\rm B}$



- minimal model: *n-p-e* matter; dominated by the density and compositional gradients (proton fraction)
- muons play a role when they emerge $\quad n \leftrightarrow p + \mu,\ x = y_e + y_\mu$

e.g. muon threshold

- pure nucleonic matter

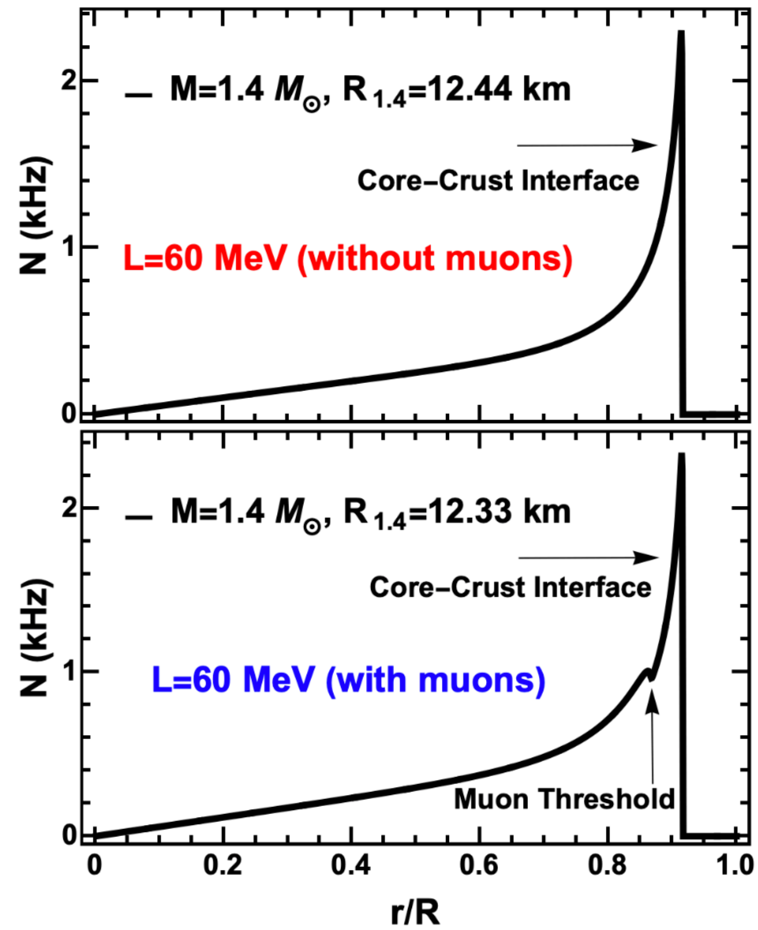
$$c_{ad}^2 - c_{eq}^2 = -\frac{n_B^2}{\mu_n}\left(\frac{\partial\tilde{\mu}}{\partial n_B}\right)_x \frac{dx}{dn_B}$$

$$\frac{\partial p}{\partial x} = n_B^2 \frac{\partial \tilde{\mu}}{\partial n_B} \qquad \tilde{\mu} = \mu_e + \mu_p - \mu_n$$

} no muons

with muons

$$c_{ad}^2 - c_{eq}^2 = -\frac{1}{\mu_n}\left(\left.\frac{\partial p}{\partial x}\right|_{n_B,y}\frac{dx}{dn_B} + \left.\frac{\partial p}{\partial y}\right|_{n_B,x}\frac{dy}{dn_B}\right)$$



- change in the sound speed **difference** signals the appearance of a new species

arXiv:2101.06349

Hybrid stars

hadronic matter only (standard)

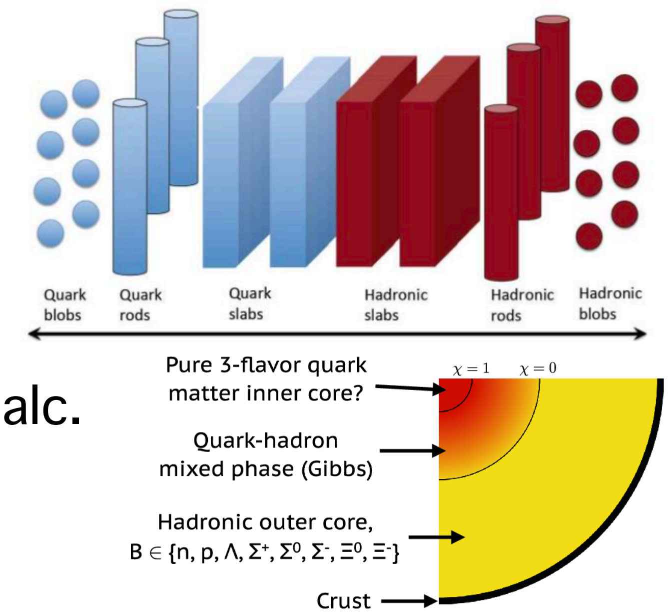
known up to saturation → **1st-order -> quark**

sharp boundary - Maxwell

mixed phase - Gibbs (geometrically separated)

crossover (quarkyonic)

- masquerade problem: likely indistinguishable through observations that constrain M-R only
- e.g. crossover “KW” EoS motivated by lattice calc.
- 1st-OPT: mixed phase (Gibbs) is favored if the hadron/quark surface tension is small

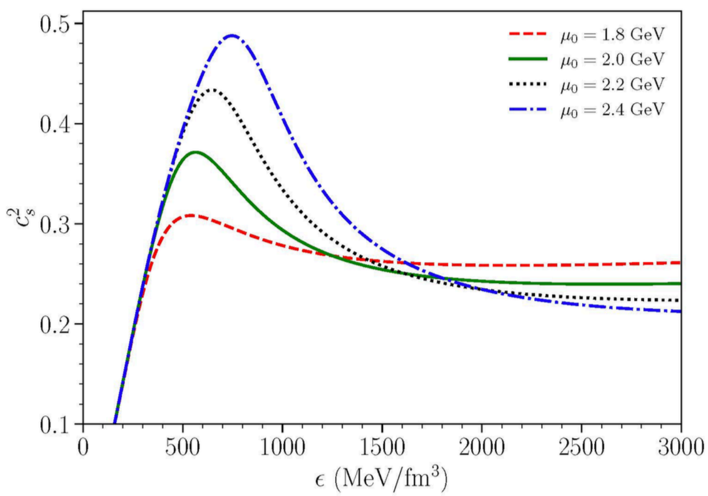


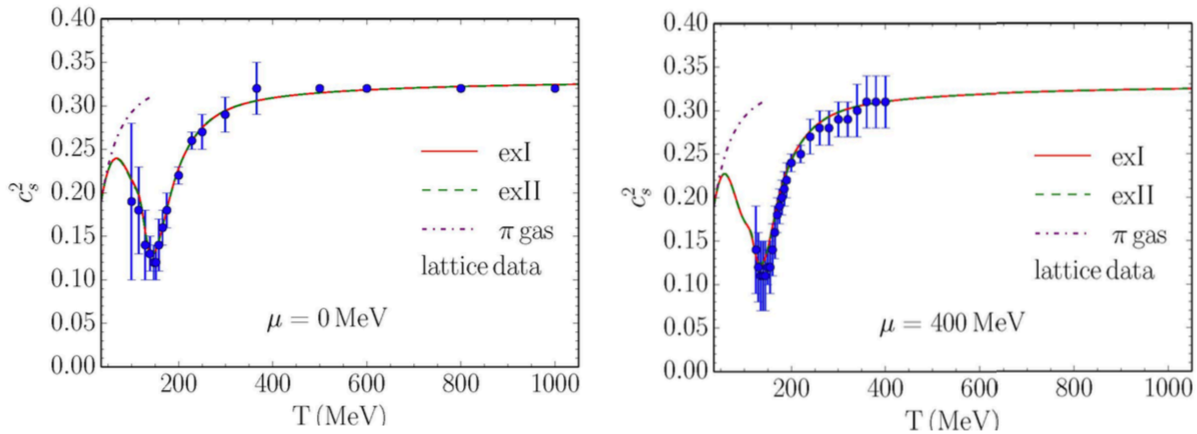
Crossover matter

- Kapusta-Welle approach: switching function of baryon chemical potential (arXiv:2103.16633)

$$P_B = (1 - S)P_H + SP_Q$$

$$S = \exp\left[-(\mu_0/\mu)^4\right]$$





$$P(T,\mu) = SP_{\rm pQCD} + (1-S)P_{\rm hadron} \qquad S = \exp\left[-\frac{1}{(T/T_0)^n + (\mu/3\pi T_0)^n}\right]$$

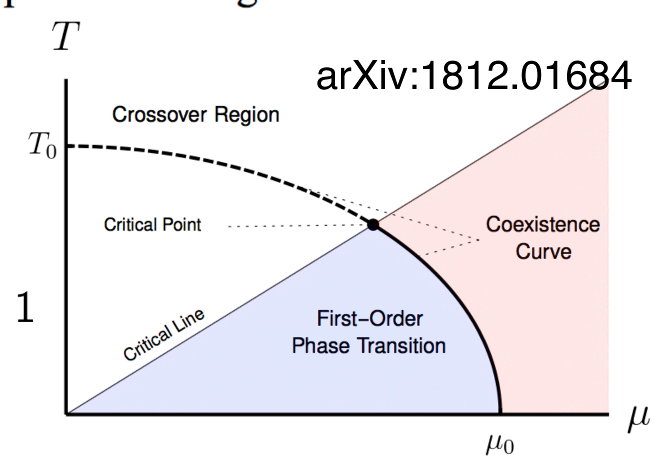
Albright, Kapusta & Young 2015.

- analogy: lattice QCD shows a crossover at finite temperature

$$S = 1/2$$

$$\left(\frac{T}{T_0}\right)^2 + \left(\frac{\mu}{\mu_0}\right)^2 = 1$$

arXiv:1812.01684



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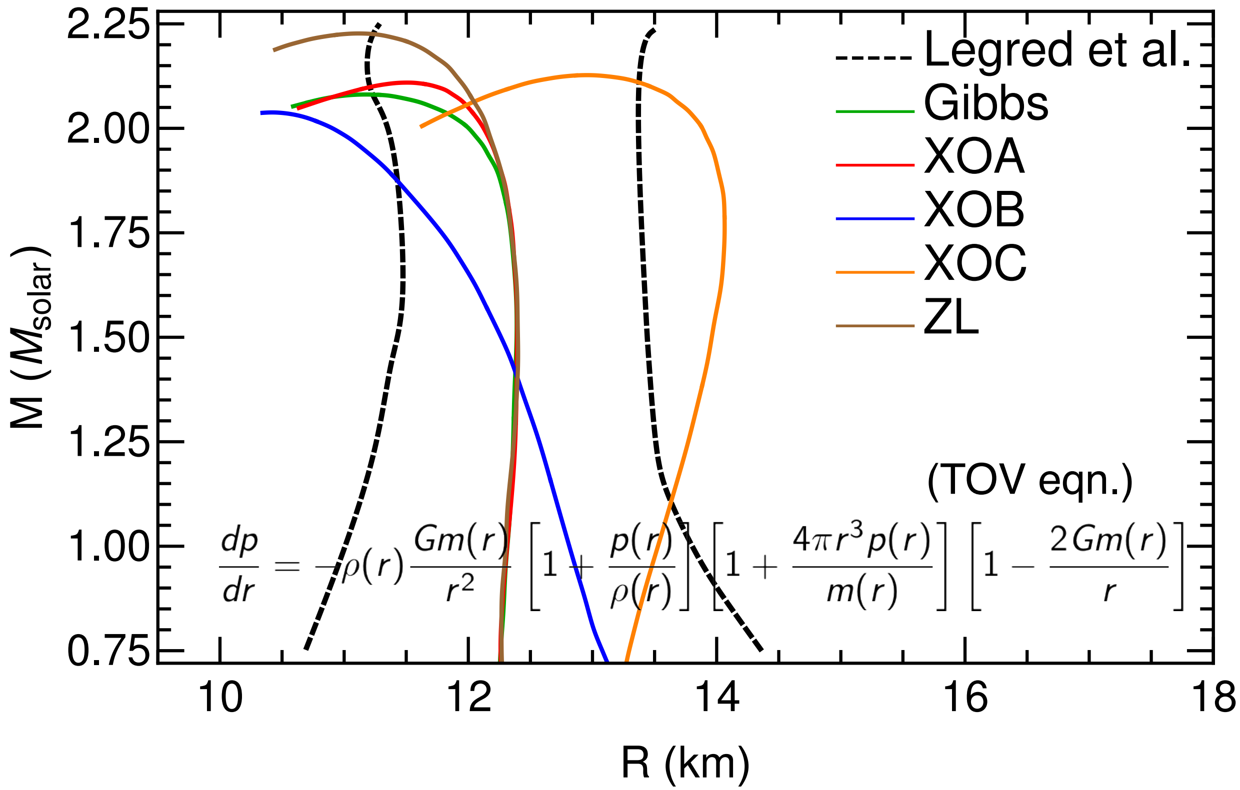
$$S = \exp\left[-(\mu_0/\mu)^4\right]$$

$$\mu = \frac{n_n\mu_n + n_p\mu_p}{n_n + n_p}$$

$$n_i = \frac{\partial P}{\partial \mu_i}\bigg|_{\mu_j}$$



- our work: construct Gibbs mixed phase and crossover using
  ZL (nucleonic) + vMIT (quark) + KW model parameters

arXiv:2109.14091

Two sound speeds

- adiabatic: start with the unconstrained system -> compute partial derivatives -> evaluate quantities at beta-equilibrium

$$(i = n, p, u, d, s, e, \mu)$$

$$c_{ad}^2(n_B, y_i) = \left(\frac{\partial p}{\partial n_B}\right)_{y_i} \left(\frac{\partial \varepsilon}{\partial n_B}\right)_{y_i}^{-1}$$

enforce beta-equilibrium

↓

$$y_i \to y_{i,\beta}(n_B)$$

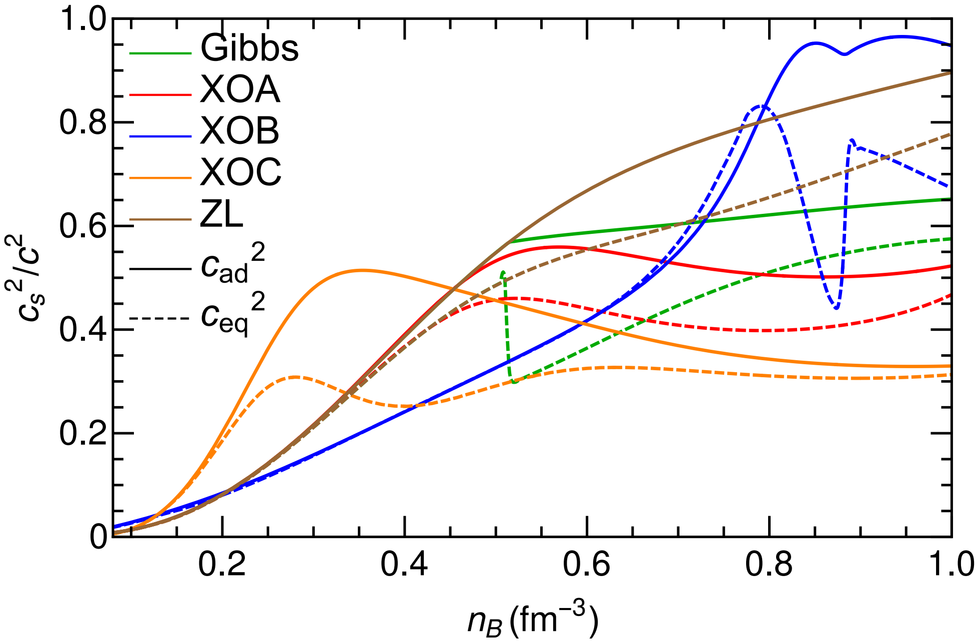
$$c_{ad,\beta}^2(n_B) = c_{ad}^2\left[n_B, y_{i,\beta}(n_B)\right]$$

$$\mu_n = \mu_p + \mu_e;\ \mu_e = \mu_\mu;\ \mu_d = \mu_s$$

$$\mu_n = \mu_u + 2\mu_d;\ \mu_p = 2\mu_u + \mu_d$$

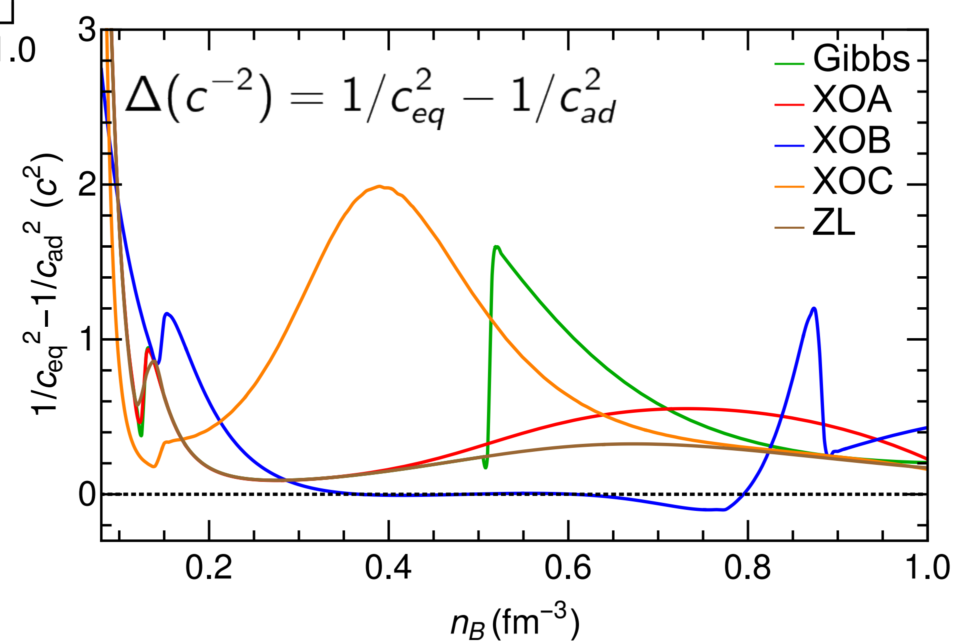
- equilibrium sound speed $c_{eq}^2(n_B) \equiv \left(\frac{dp}{d\varepsilon}\right)_\beta = \left(\frac{dp}{dn_B}\right)_\beta \left(\frac{d\varepsilon}{dn_B}\right)_\beta^{-1}$

Sound speed profiles



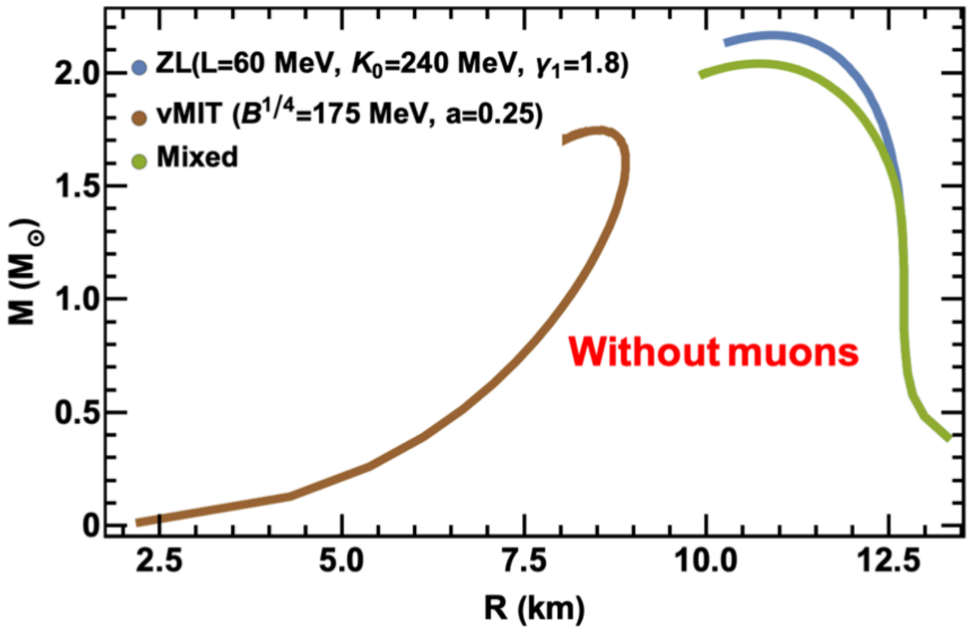
- nucleonic only (ZL) - both increase monotonically
- admixtures of nucleons and quarks (Gibbs or crossover) induce non-monotonic behavior
- $c_{ad}^2 > c_{eq}^2$ for all densities except XOB

- difference of the inverses of the adiabatic and equilibrium sound speeds
- reflects substantial changes in the particle fraction

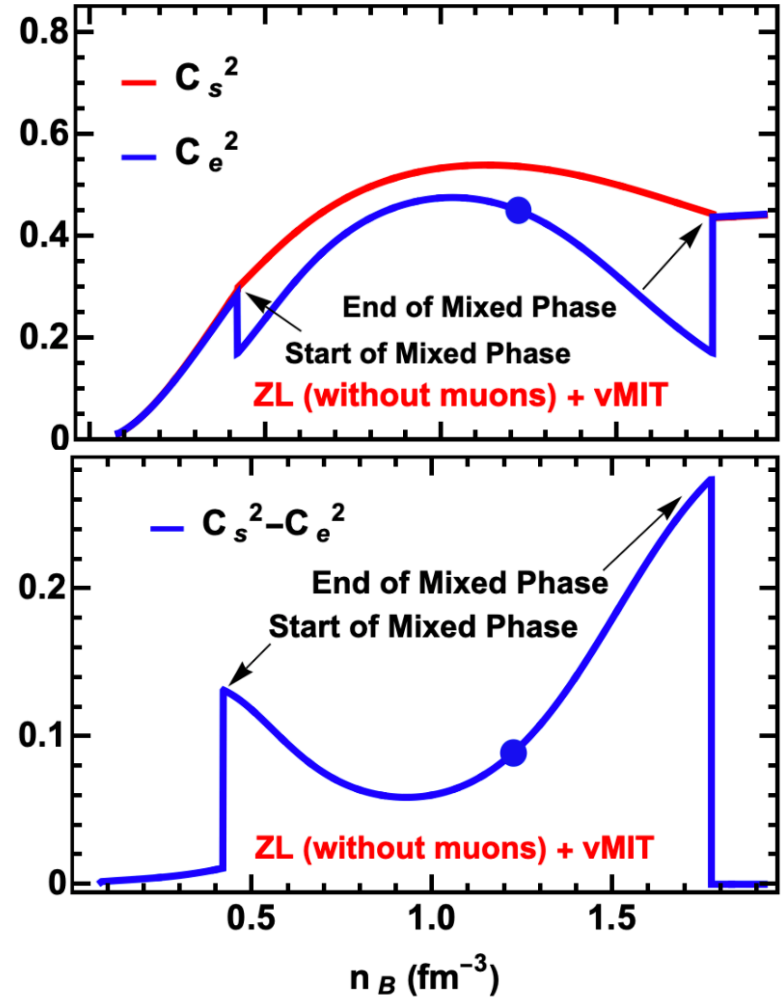


1st-OPT mixed phase (Gibbs)

$$\frac{1}{c_{ad}^{*2}} = \frac{1-\chi}{c_{ad,H}^2} + \frac{\chi}{c_{ad,Q}^2}$$

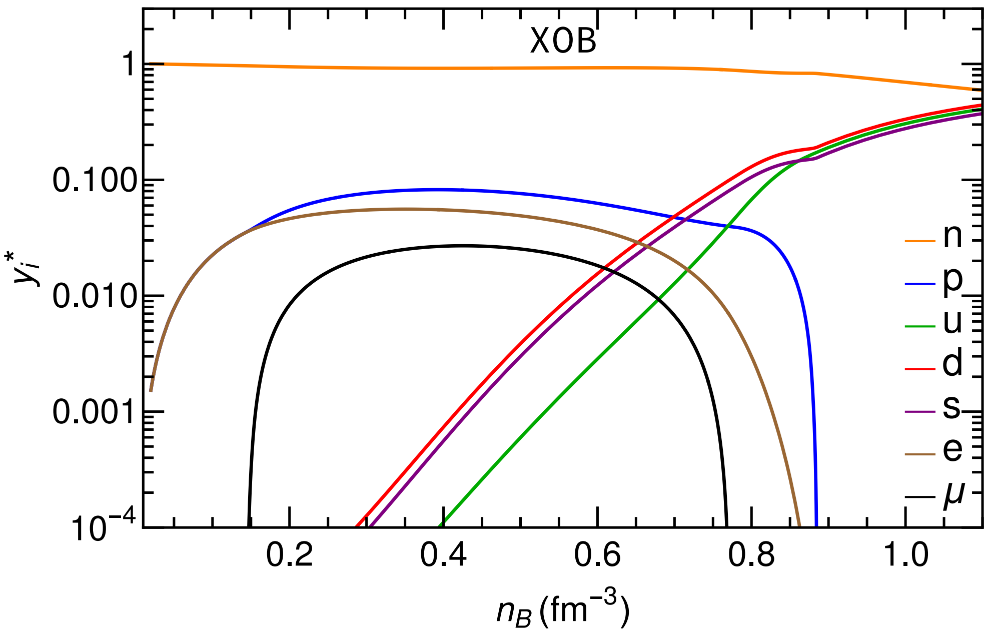


$$\frac{1}{c_{eq}^{*2}} = \frac{1-\chi}{c_{eq,H}^2} + \frac{\chi}{c_{eq,Q}^2} + (\varepsilon_Q - \varepsilon_H)\frac{d\chi/dn_B}{dp/dn_B}$$

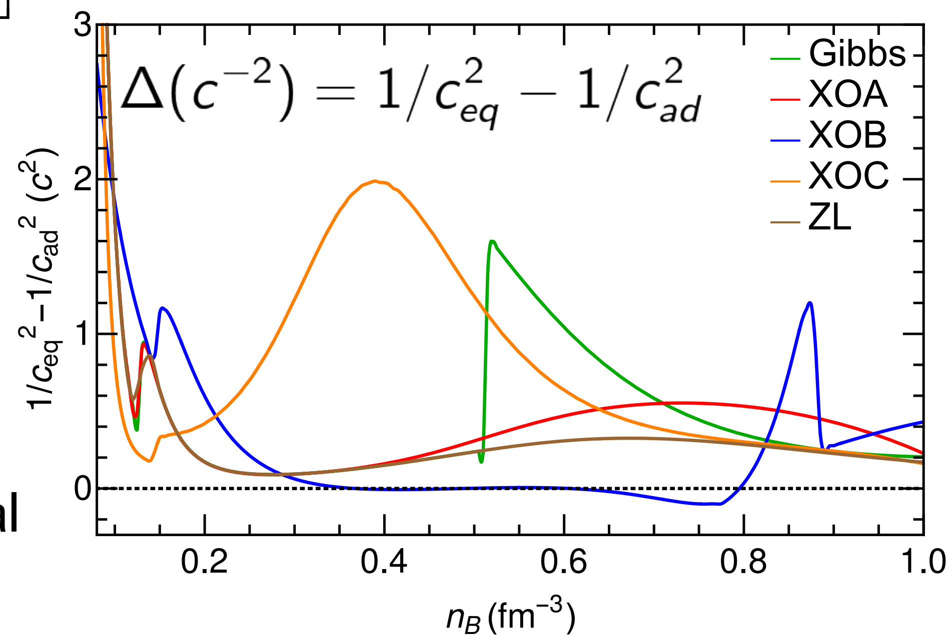


arXiv:2101.06349

Sound speed and composition



- muons set in ~nsat
- peak ~5.5 nsat: muons and protons disappear
- instability to convection - unphysical region



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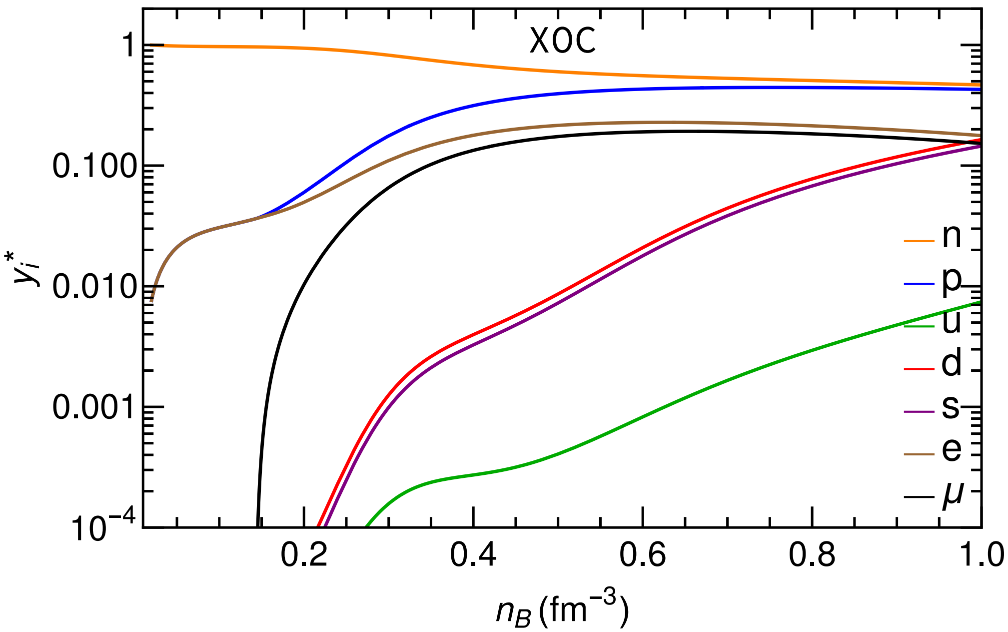
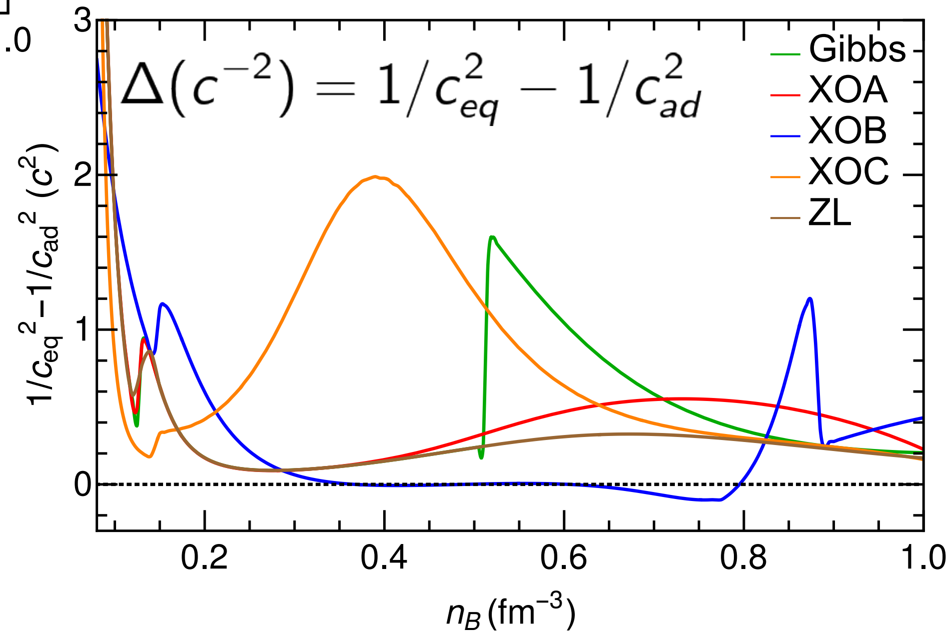


- muons set in ~nsat
- peak ~2.5 nsat: inflection points in quark and neutron fraction
- too stiff at low densities - radii too large



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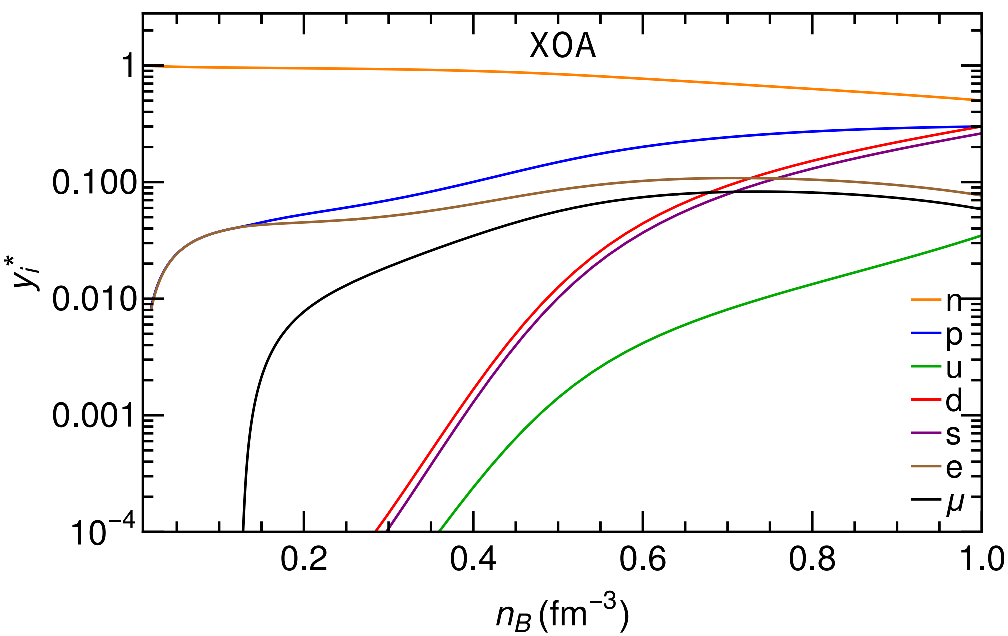
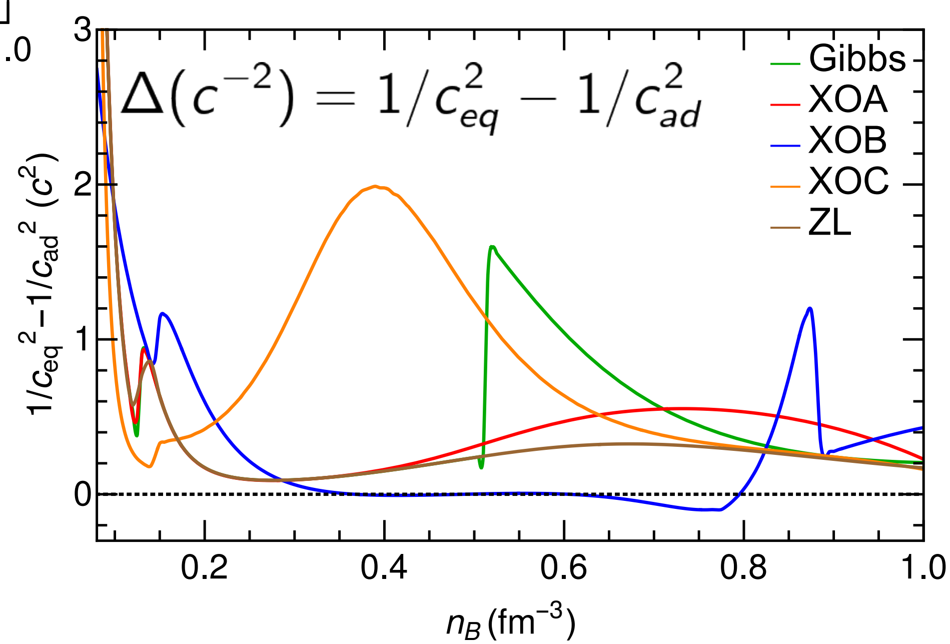
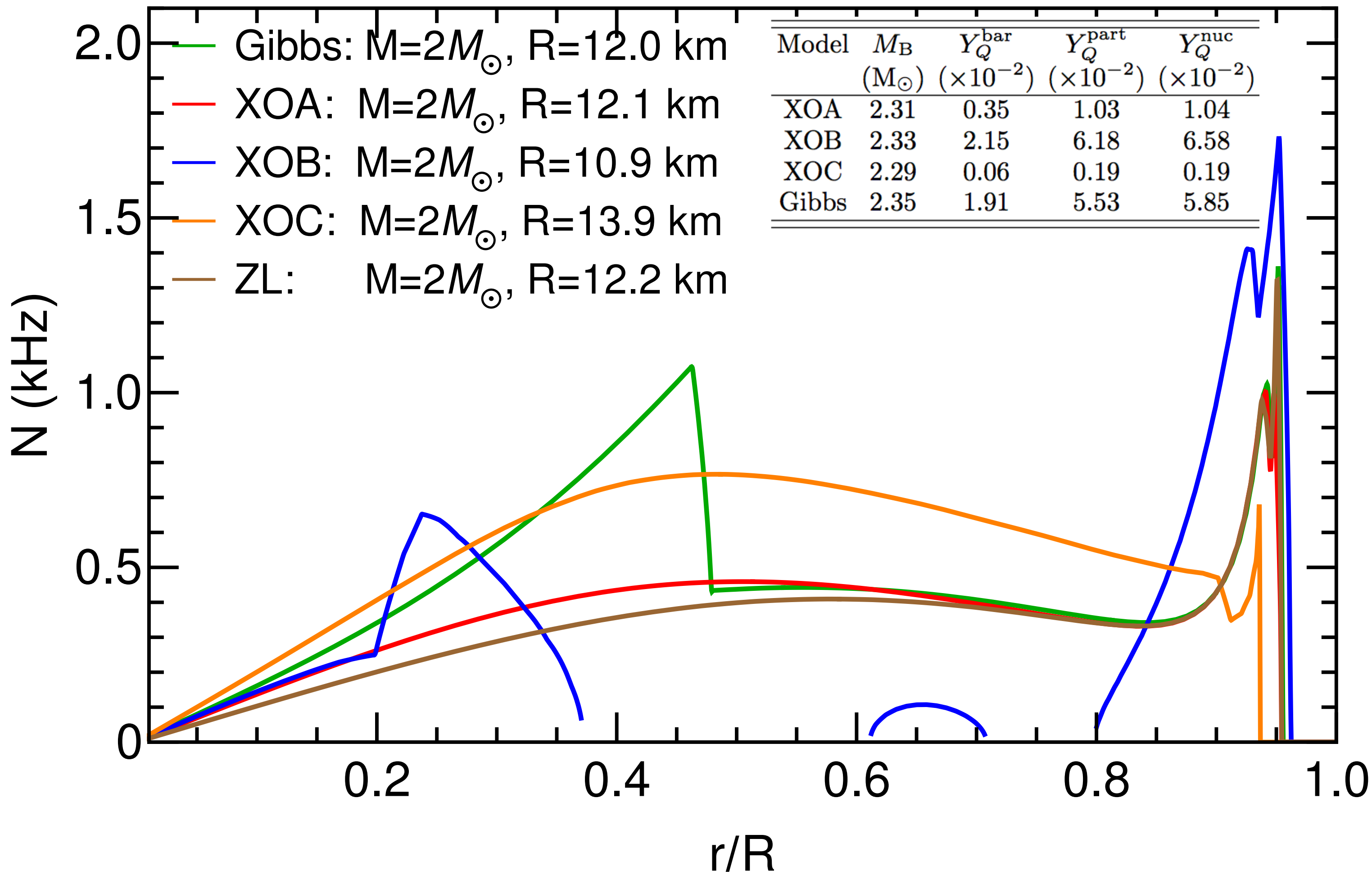




- muons set in ~nsat
- "good" crossover model, very similar to ZL
- Gibbs peak ~2.5 nsat: onset of mixed phase

Brunt–Väisälä in a hybrid star



Model	M_B ($M_\odot$)	Y_Q^{bar} ($\times 10^{-2}$)	Y_Q^{part} ($\times 10^{-2}$)	Y_Q^{nuc} ($\times 10^{-2}$)
XOA	2.31	0.35	1.03	1.04
XOB	2.33	2.15	6.18	6.58
XOC	2.29	0.06	0.19	0.19
Gibbs	2.35	1.91	5.53	5.85

arXiv:2109.14091

Global g-mode frequency

$$\frac{dU}{dr} = \frac{g}{c_{ad}^2} U + e^{\lambda/2} \left[\frac{l(l+1)e^{\nu}}{\omega^2} - \frac{r^2}{c_{ad}^2} \right] V$$

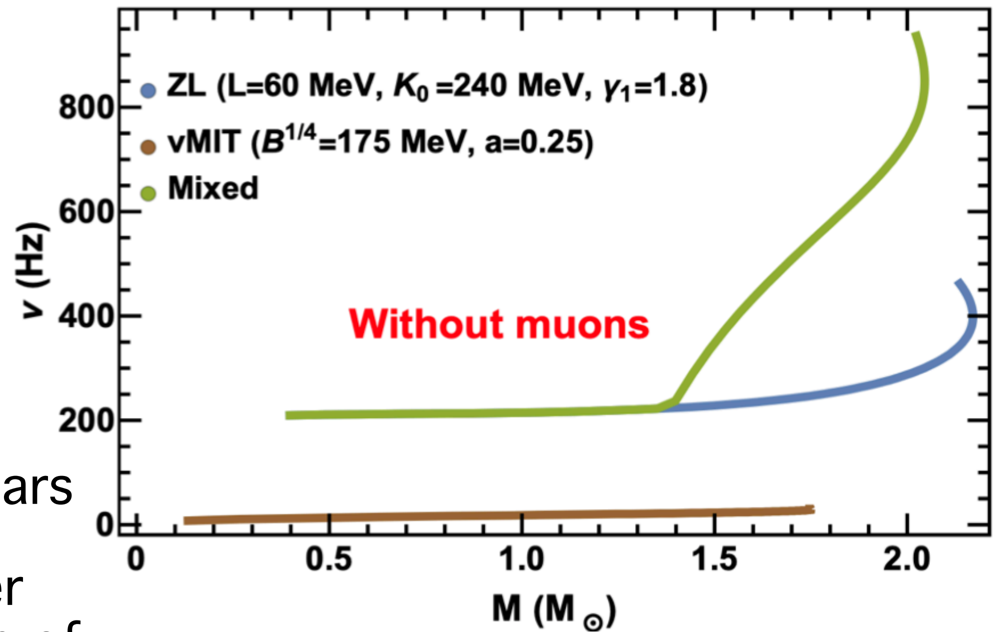
$$\frac{dV}{dr} = e^{\lambda/2-\nu} \left(\frac{\omega^2 - N^2}{r^2} \right) U + g\Delta(c^{-2}) V$$

$$\Delta(c^{-2}) = 1/c_{eq}^2 - 1/c_{ad}^2$$

$$\nu_g = \omega/(2\pi)$$

- $\nu_g \gtrsim 600\,\text{Hz}$ only for hybrid stars
- signature of exotic phases - higher frequency indicates larger fraction of quark matter

- most distinct feature of Gibbs
- mixed phase onset -> peak in local BV frequency -> kink in global g-mode frequency



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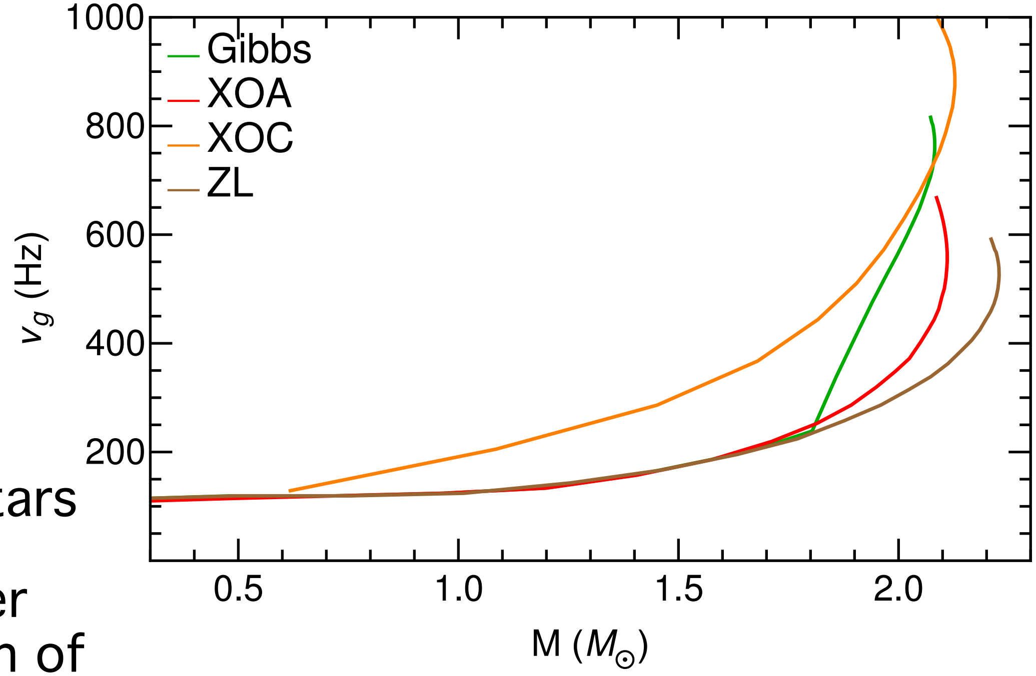


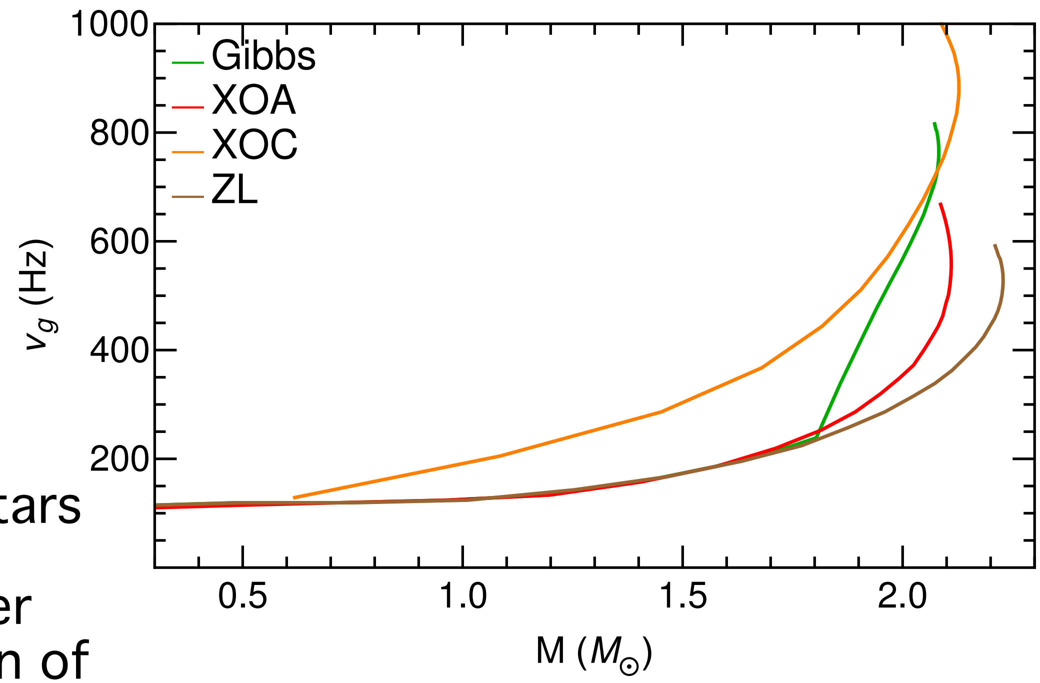
arXiv:2109.14091

Global g-mode frequency

- comparison with other works

Authors [Ref.]	Core Composition	M [$M_\odot$]	$f_g = \omega_g/(2\pi)$ [kHz]
Reisenegger & Goldreich [17]	npe	1.405	0.215
Lai [20]	npe	1.4	0.073
Kantor & Gusakov [29]	npe	1.4	0.13
Kantor & Gusakov [29]	$npe\mu$	1.4	0.19
Kantor & Gusakov [29]	$npe\mu$(SF)	1.4	0.46
Dommes & Gusakov [30]	$npe\mu\Lambda$(SF)	1.634	0.742
Yu & Weinberg [33]	$npe\mu$	1.4	0.13
Yu & Weinberg [33]	$npe\mu$(SF)	2.0	0.45
Rau & Wasserman [34]	$npe\mu$(SF)	2.0	0.45

- most distinct feature of Gibbs
- mixed phase onset -> peak in local BV frequency -> kink in global g-mode frequency



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- $\nu_g \gtrsim 600$ Hz only for hybrid stars
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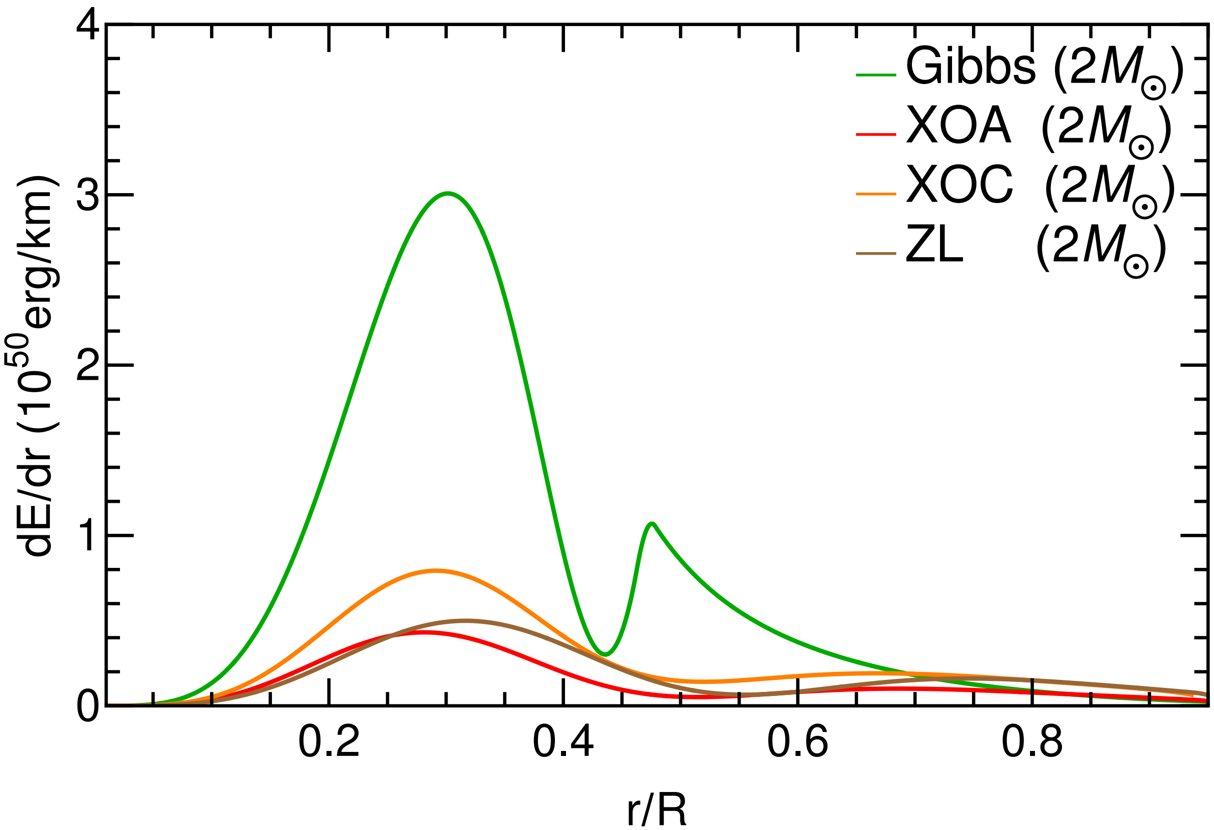
Mode energies and tidal forcing



$$\frac{dE}{dr} = \frac{\omega^2 r^2}{2}(\varepsilon + p)e^{(\lambda-\nu)/2}\left[\xi_r^2 e^{\lambda} + l(l+1)\xi_h^2\right]$$

$$U = r^2 e^{\lambda/2}\xi_r, \text{ radial}$$

$$V = \omega^2 r \xi_h, \text{ tangential}$$

- energy per unit radial distance contained in the oscillatory motion

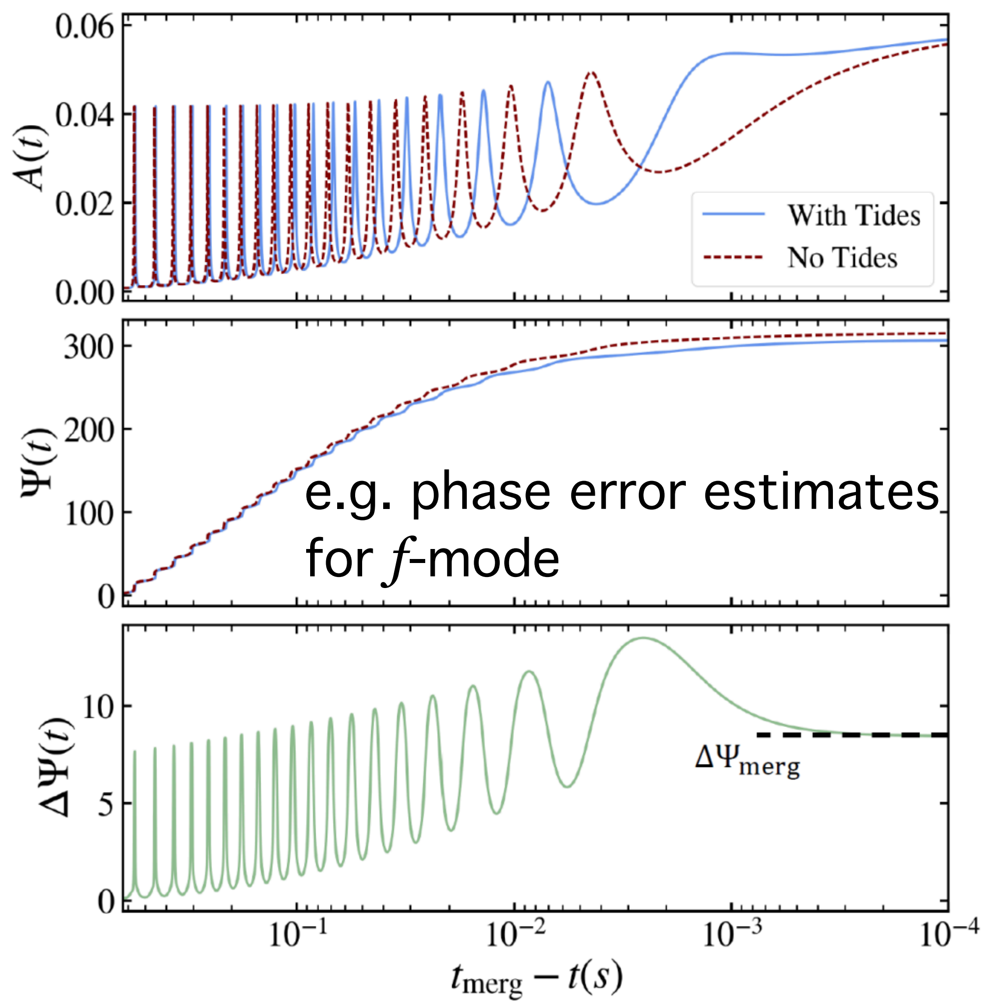
Estimated impact on the GW waveform

Vick & Lai (2019)

$$\left|\frac{\Delta E}{E}\right| \simeq 2.3 \times 10^{-3},\ \mathrm{NS}$$
$$\simeq 5.9 \times 10^{-3},\ \mathrm{HS\ (Gibbs)}$$
$$\simeq 2 \times 10^{-9},\ \mathrm{SQS}$$

- resonant excitation leads to phase shift later in the inspiral (at higher frequencies); shorter duration of accumulation

$$\Delta\phi \simeq 0.8,\ \mathrm{NS}$$
$$\simeq 0.45,\ \mathrm{HS\ (Gibbs)}$$
$$\simeq 6 \times 10^{-4},\ \mathrm{SQS}$$



Conclusions

- **Stellar oscillation modes carry imprints of the phase of matter interior through resonant excitation frequencies**

- ***g*-modes can probe stratification: mixed phase/crossover/crust of neutron stars**

- **Compared to bulk properties such as masses and radii, oscillation modes are more sensitive to composition, but may need continuous GW sources and/or large amplitudes**

- **Detection of oscillation modes is worth pursing with improved sensitivity and more detectors as in 3G network**

THANK YOU!

Q & A